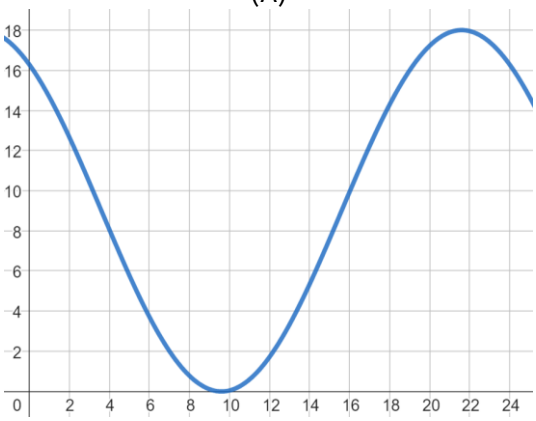
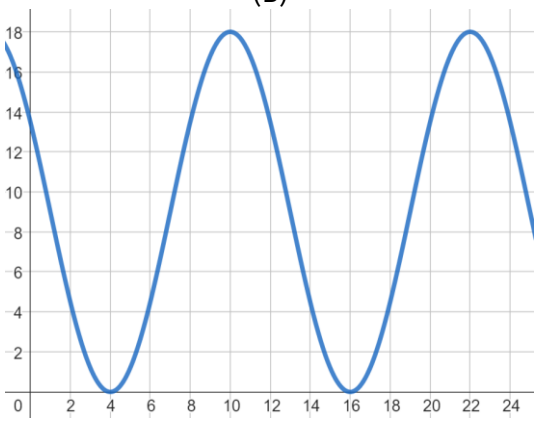
Sinusoidal Functions in Context Homework (3.6+3.7)

1. At 4:00 AM, it is low tide on Ft Lauderdale beach. At that point, the tidal height is only 8 feet. Six hours later, the first high tide of the day occurs. At that point, the tidal height is 18 feet.
   a. Which of the following graphs represents the tidal height, in feet, as a function of time, in hours since midnight?

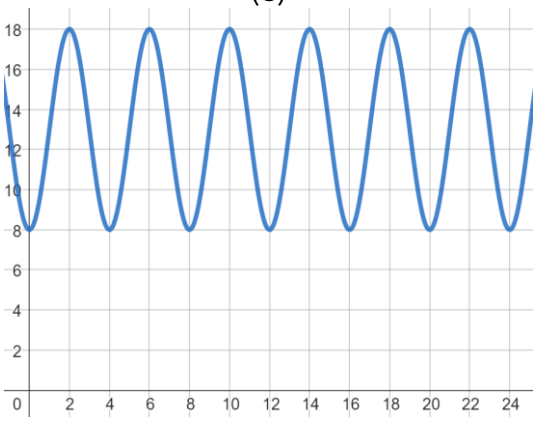
(A)



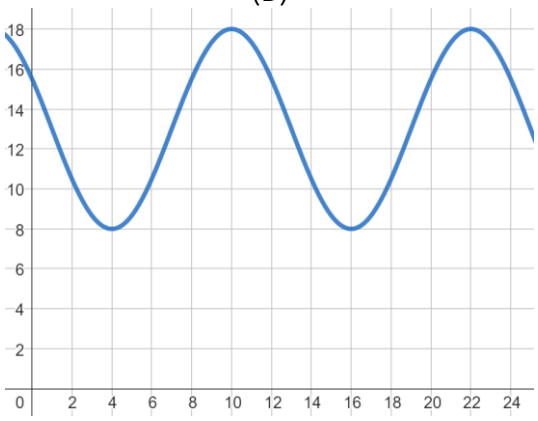
(B)



(C)



(D)



   b. Which of the following equations represents the tidal height, in feet, as a function of time, in hours, since midnight?

   (A) $h(t) = 5\cos\left(\frac{\pi}{12}(t-1)\right) + 13$

   (B) $h(t) = 5\cos\left(\frac{\pi}{6}(t-4)\right) + 13$

   (C) $h(t) = 5\sin\left(\frac{\pi}{12}(t-4)\right) + 13$

   (D) $h(t) = 5\sin\left(\frac{\pi}{6}(t-7)\right) + 13$

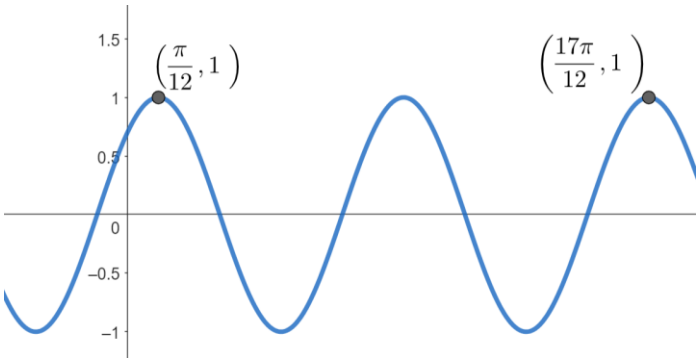
2. To the right is the graph of $y = \cos\big(b(x+c)\big)$.
   a. Find the value of b.
   b. Which one of the following is a possible value of c?

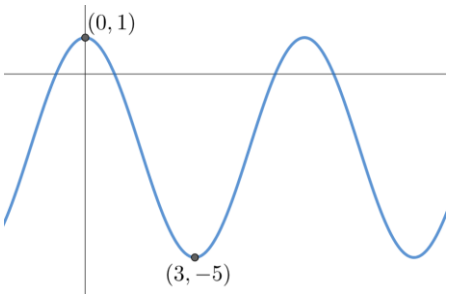
   (A) $-\frac{\pi}{12}$

   (B) $-\frac{\pi}{36}$

   (C) $\frac{\pi}{12}$

   (D) $\frac{\pi}{36}$





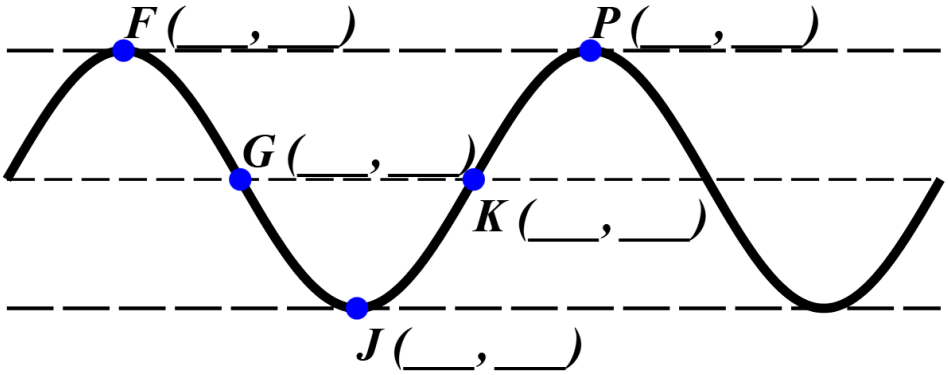
3. To the left is the graph of $a\cos(bx)+c$. Find the values of a, b, and c.

4. Consider the function $y=\cos(3x)$. Sketching this function is not necessary but nonetheless may be helpful in answering both parts of this question.
   a. At which of the following x-values does this function have relative minima? Select all that apply.

$\frac{\pi}{3}$	$-\frac{\pi}{2}$	$\frac{2\pi}{3}$	0	$\frac{3\pi}{2}$
π	$\frac{7\pi}{3}$	$\frac{7\pi}{6}$	$\frac{11\pi}{6}$	$-\frac{\pi}{3}$

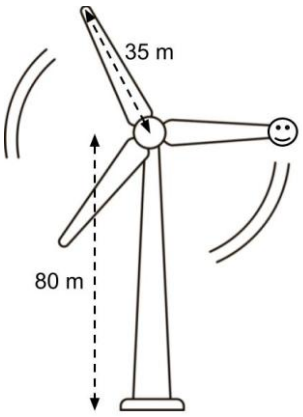
   b. Which one of the following represents ALL x-values at which this function has relative minima?
   (A) $\frac{\pi}{3}+\frac{2\pi}{3}n, n\in\mathbb{Z}$
   (B) $\frac{2\pi}{3}n, n\in\mathbb{Z}$
   (C) $\frac{\pi}{3}n, n\in\mathbb{Z}$
   (D) $\frac{\pi}{6}+\frac{\pi}{3}n, n\in\mathbb{Z}$

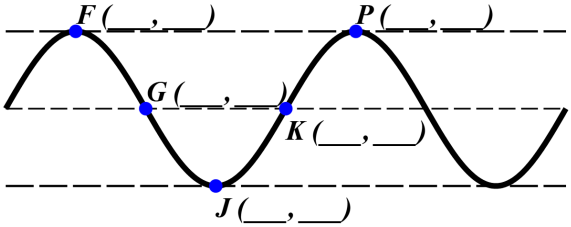
5. Sally is see-sawing. At time $t=0$, she is 40 inches off the ground. It takes 2 seconds for her to fall to a height of 4 inches off the ground and another 2 seconds for her to rise back up to 40 inches off the ground. The process then repeats for 5 minutes of continuous see-sawing. Let $h(t)$ be a sinusoidal function modelling Sally's height off the ground at time t (in seconds). Find the coordinates of the five labelled points on the graph of $h(t)$



Which one of the following would be a good choice of $h(t)$?
(A) $18\cos\left(\frac{\pi}{2}t\right)+22$
(B) $18\sin\left(\frac{\pi}{2}t\right)+22$
(C) $18\cos(\pi t)+22$
(D) $18\sin(\pi t)+22$

6. Rose is a bug on the edge of a wind turbine. Every minute, the wind turbine turns a half of a revolution. The middle of the turbine is 80 meters high, and the blades of the turbine are 35 meters in length. Rosie is at the tip of one of the blades. At time $t = 0$, Rosie is directly to the right of the center, as is shown in the diagram. The turbine turns in a clockwise direction. Let $h(t)$ be a function that models Rose's height above the ground in meters for t given in minutes. Find coordinates for the five labelled points on the below graph of $h(t)$.





Which one of the following would be a good choice of $h(t)$?

(A) $35 - 80\sin(\pi t)$
(B) $80 - 35\sin(\pi t)$
(C) $35 + 80\sin(\pi t)$
(D) $80 + 35\sin(\pi t)$

7. As Jorge swings on a trapeze, his height off the ground can be modelled by $h(t) = 3.1\cos(1.427t) + 20$ where $h(t)$ is given in feet off the ground and t is given in seconds since he began swinging. What is Jorge's maximum height off the ground? What is Jorge's minimum height off the ground?

8. At exactly 12:00 PM, Lucy gets on a Ferris wheel with diameter 100 feet. Being a big fan of Ferris wheels, she stays on for eighty minutes, long enough that the Ferris wheel makes exactly 16 full rotations. Let $h(t)$ represent Lucy's height t minutes after noon measured in feet above her initial height. Which of the following would be the most appropriate model for $h(t)$?

(A) $50 - 50\cos\left(\frac{t}{5}\right)$
(B) $50 - 50\cos\left(\frac{2\pi}{5}t\right)$
(C) $50 - 50\cos(16t)$
(D) $50 - 50\cos(10\pi t)$

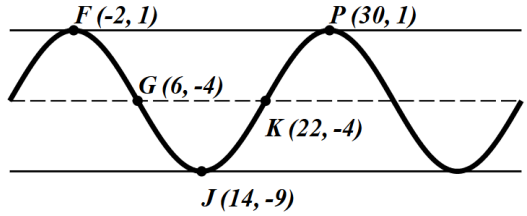
9. Below are select values of the function $f(x)$. The graph of $g(x)$ is the image of the graph of $f(x)$ after the following transformations in the following order: a vertical translation by -3 units, a reflection over the x-axis, a horizontal dilation by a factor of $\frac{1}{2}$.

x	0	2	4	6	8
$f(x)$	1	3	-5	-6	-10

a. Write an expression for $g(x)$ in terms of $f(x)$

b. Find $g(4)$

10. Below two full cycles of the periodic function $h(x)$ are shown. Five points are labelled. No axes are presented. The function h can be written in the form $h(x) = a\sin\big(b(x+c)\big) + d$.



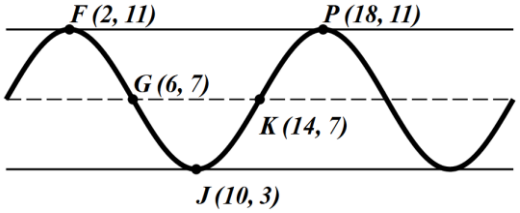
a. Find values of constants a, b, c, and d.

b. Let t_1 be the x-coordinate of point G and t_2 be the x-coordinate of point J. On the interval (t_1, t_2) which of the following is true about h?

(A) h is positive and increasing
(B) h is positive and decreasing
(C) h is negative and increasing
(D) h is negative and decreasing

c. On the interval (t_1, t_2), describe the concavity of $h(x)$ and determine whether the rate of change of h is increasing or decreasing.

11. Below two full cycles of the periodic function $h(x)$ are shown. Five points are labelled. No axes are presented. The function h can be written in the form $h(x) = a\cos\big(b(x+c)\big) + d$.



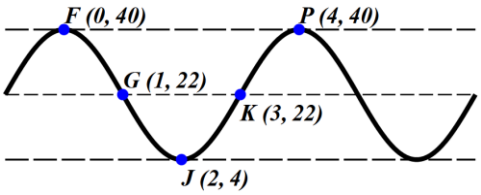
a. Find values of constants a, b, c, and d.

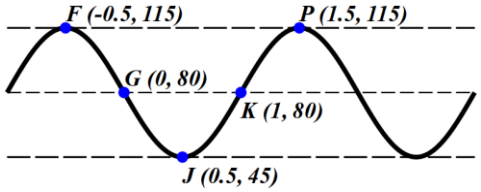
b. Let t_1 be the x-coordinate of point K and t_2 be the x-coordinate of point P. On the interval (t_1, t_2) which of the following is true about h?

(A) h is positive and increasing
(B) h is positive and decreasing
(C) h is negative and increasing
(D) h is negative and decreasing

c. On the interval (t_1, t_2), describe the concavity of $h(x)$ and determine whether the rate of change of h is increasing or decreasing.

Sinusoidal Functions in Context HW Answers

1.
   a. D
   b. D
2.
   a. 3
   b. A
3. $a = 3$, $b = \frac{\pi}{3}$, and $c = -2$
4.
   a. $-\frac{\pi}{3}, \frac{\pi}{3}, \pi$, and $\frac{7\pi}{3}$
   b. A
5.


   (A) $18\cos\left(\frac{\pi}{2}t\right) + 22$
6.


   (B) $80 - 35\sin(\pi t)$
7. Maximum: 23.1 feet
   Minimum: 16.9 feet
8. B
9.
   a. $g(x) = -f(2x) + 3$
   b. $g(4) = 13$
10. Part a: Any one of the following sets of values would earn full credit.

One correct answer:	$a = 5$	$b = \frac{\pi}{16}$	$c = -22$	$d = -4$
Also correct:	$a = 5$	$b = \frac{\pi}{16}$	$c = 10$	$d = -4$
Also correct:	$a = -5$	$b = \frac{\pi}{16}$	$c = -6$	$d = -4$

    Part b: (D) h is negative and decreasing
    Part c: On the interval (t_1, t_2), the graph of h is concave up. On the interval (t_1, t_2), the rate of change of h is increasing.
11. Part a: Any one of the following sets of values would earn full credit.

One correct answer:	$a = 4$	$b = \frac{\pi}{8}$	$c = -2$	$d = 7$
Also correct:	$a = 4$	$b = \frac{\pi}{8}$	$c = -18$	$d = 7$
Also correct:	$a = -4$	$b = \frac{\pi}{8}$	$c = -10$	$d = 7$

    Part b: (A) h is positive and increasing
    Part c: On the interval (t_1, t_2), the graph of h is concave down. On the interval (t_1, t_2), the rate of change of h is decreasing.